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Solutions to PS #6

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Problem 1:

$$^1S \leftarrow l=0 \quad \therefore j = l+s, \dots, |l-s| = 0.$$

$l=(2s+1) \Rightarrow s=0$

$$^3S \leftarrow l=0 \quad \therefore j = 1$$

$3=(2s+1) \Rightarrow s=1$

$$^1P \leftarrow l=1 \quad \therefore j = 1$$

$1=(2s+1) \Rightarrow s=0$

$$^3P \leftarrow l=1 \quad \therefore j = 1+1=2, 1, 1-1=0 \Rightarrow j=2, 1, 0$$

$3=2s+1 \Rightarrow s=1$

$$^2D \leftarrow l=2 \quad \therefore j = 2+\frac{1}{2} = \frac{5}{2}$$

$2=(2s+1) \Rightarrow s=\frac{1}{2}$

) no integer steps in between

$$2-\frac{1}{2} = \frac{3}{2}$$

$$^4D \leftarrow l=2 \quad \therefore j = 2+\frac{3}{2} = \frac{7}{2} \quad \therefore j = \frac{7}{2}, \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$$

$4=(2s+1) \Rightarrow s=\frac{3}{2}$

$$\begin{matrix} \frac{5}{2} \\ \frac{3}{2} \\ \frac{1}{2} \\ 2-\frac{3}{2} = \frac{1}{2} \end{matrix}$$

Problem 2

(n s n' s): electrons in different n levels \therefore their spatial wavefunctions can be symmetric or antisymmetric $\Rightarrow s=0$ or 1 (symmetric and antisymmetric spin configurations for 2 spin $\frac{1}{2}$ particles).

$\overrightarrow{\text{symmetric}} \quad \overrightarrow{\text{antisymmetric}}$

$l_1=0, l_2=0 \quad \therefore l_{\text{tot}}=0$

$$\therefore ^1S_0, ^3S_1$$

(n s n' p): Again electrons in different n levels \therefore possible s values are 0 or 1

$\overrightarrow{l_1=0, l_2=1} \quad \therefore l_{\text{tot}}=1$

$$\therefore ^1P_0, ^3P_1$$

(n s n'd): again $s=0$ or 1
 $\ell_1=0 \quad \ell_2=2 \quad \rightarrow \ell_{\text{tot}}=2$

$$\therefore \begin{array}{c} {}^1D_2, {}^3D_3, {}^3D_1, {}^3D_2 \\ \uparrow \ell_{\text{tot}}=2 \qquad \downarrow \ell_{\text{tot}}=2 \\ s=0 \qquad \qquad \qquad s=1 \end{array}$$

(npn'p): again $s=0$ or 1
 $\ell_1=1, \ell_2=1 \quad \therefore \ell_{\text{tot}}=2, 1, 0$

$$\therefore \begin{array}{c} \ell_{\text{tot}}=2 \quad \begin{array}{c} {}^1D_2, {}^3D_3, {}^3D_2, {}^3D_1 \\ \uparrow \qquad \downarrow \\ s=0 \qquad s=1 \end{array} \\ \ell_{\text{tot}}=1 \quad {}^1P_1, {}^3P_2, {}^3P_1, {}^3P_0 \\ \ell_{\text{tot}}=0 \quad {}^3S_0, {}^3S_1 \end{array}$$

Problem 3

(a) Particles have spin 0: \therefore we need to symmetrize their total wavefunction.
 Since there is no spin wavefunction, the spatial wavefunction must be symmetric.

$$\therefore |\Psi\rangle = \frac{1}{\sqrt{2}} (|1n\rangle, |1m\rangle_2 + |1m\rangle, |1n\rangle_2)$$

As stated in class, the problem has been changed to computing the expectation value of $V(x_1, x_2)$.

$$V(x_1, x_2) = \lambda (x_1 - x_2)^2 = \lambda (x_1^2 + x_2^2 - 2x_1 x_2)$$

$$x_1 = \left(\frac{\hbar}{2m\omega}\right)^{1/2} (a_1^+ + a_1), \quad x_2 = \left(\frac{\hbar}{2m\omega}\right)^{1/2} (a_2^+ + a_2)$$

$$x_1^2 = \frac{\hbar}{2m\omega} (a_1^{+2} + a_1^2 + a_1^+ a_1 + a_1 a_1^+) ; \quad x_2^2 = \frac{\hbar}{2m\omega} (a_2^{+2} + a_2^2 + a_2^+ a_2 + a_2 a_2^+)$$

$$E' = \langle \Psi | V | \Psi \rangle = \langle \Psi | x_1^2 | \Psi \rangle + \langle \Psi | x_2^2 | \Psi \rangle - 2 \langle \Psi | x_1 x_2 | \Psi \rangle$$

$$a^+ |n\rangle = \sqrt{n+1} |n+1\rangle$$

$$a |n\rangle = \sqrt{n} |n-1\rangle$$

$$\cdot \langle \Psi | x_1^2 | \Psi \rangle = \frac{1}{\sqrt{2}} \left[\langle n_1 | \langle m_1 |_2 + \langle m_1 | \langle n_1 |_2 \right] \frac{\hbar}{2m\omega} (a_1^{+2} + a_1^2 + a_1^+ a_1 + a_1 a_1^+) \frac{1}{\sqrt{2}} [|1n\rangle, |1m\rangle_2 + |1m\rangle, |1n\rangle_2]$$

$$= \frac{\hbar}{4m\omega} \left[\langle n_1 | \langle m_1 |_2 + \langle m_1 | \langle n_1 |_2 \right] \underbrace{[\sqrt{n+1} \sqrt{n+2} |n+2\rangle, |1m\rangle_2 + \sqrt{n+1} \sqrt{n+2} |n+2\rangle, |1n\rangle_2]}_{a^+ |1\Psi\rangle}$$

$$+ \underbrace{\sqrt{n\sqrt{n+1}} |n-2\rangle, |1m\rangle_2 + \sqrt{n\sqrt{n+1}} |n-2\rangle, |1n\rangle_2 + (i\sqrt{n}) (n, |1m\rangle_2 + (n+1)|1m\rangle, |1n\rangle_2 + (n+1)|1n\rangle, |1m\rangle_2 + (n+1)|1n\rangle, |1n\rangle_2)}_{a |1\Psi\rangle} + \underbrace{(n+1) |1n\rangle, |1m\rangle_2 + (n+1) |1m\rangle, |1n\rangle_2 }_{a, a^+ |1\Psi\rangle}$$

$$\begin{aligned}
\Rightarrow \langle \psi | x_1^2 | \psi \rangle &= \frac{\hbar}{4m\omega} [n + n+1 + m + m+1] = \frac{\hbar}{4m\omega} [2(n+m) + 2] = \frac{\hbar}{2m\omega} (n+m+1) \\
\cdot \langle \psi | x_2^2 | \psi \rangle &= \frac{1}{\sqrt{2}} [\langle nl_1 \rangle \langle ml_2 \rangle + \langle ml_1 \rangle \langle nl_2 \rangle] \frac{\hbar}{2m\omega} (a_1^{+2} + a_2^{+2} + a_1^+ a_2 + a_2 a_1^+) \frac{1}{\sqrt{2}} [\langle n \rangle \langle m \rangle_2 + \langle m \rangle \langle n \rangle_2] \\
&= \frac{\hbar}{4m\omega} (\langle nl_1 \rangle \langle ml_2 \rangle + \langle ml_1 \rangle \langle nl_2 \rangle) [\sqrt{n+1}\sqrt{m+2} \langle n \rangle \langle m+2 \rangle_2 + \sqrt{m+1}\sqrt{n+2} \langle m \rangle \langle n+2 \rangle_2 + \sqrt{n}\sqrt{m-1} \langle n \rangle \langle m-2 \rangle_2 \\
&\quad + \sqrt{n+1}\sqrt{n-1} \langle m \rangle \langle n-2 \rangle_2 + m \langle n \rangle \langle m \rangle_2 + n \langle n \rangle \langle m \rangle_2 + (n+1) \langle n \rangle \langle m \rangle_2 + (n+1) \langle m \rangle \langle n \rangle_2] \\
&= \frac{\hbar}{4m\omega} [m+n+m+1+n+1] = \frac{\hbar}{2m\omega} (n+m+1) \\
\cdot \langle \psi | x_1 x_2 | \psi \rangle &= \frac{1}{\sqrt{2}} [\langle nl_1 \rangle \langle ml_2 \rangle + \langle ml_1 \rangle \langle nl_2 \rangle] \frac{\hbar}{2m\omega} \underbrace{(a_1^+ + a_1)(a_2^+ + a_2)}_{\sqrt{2}} [\langle n \rangle \langle m \rangle_2 + \langle m \rangle \langle n \rangle_2] \\
&= \frac{\hbar}{4m\omega} [\langle nl_1 \rangle \langle ml_2 \rangle + \langle ml_1 \rangle \langle nl_2 \rangle] [\underbrace{\sqrt{n+1}\sqrt{m+1} (\langle n+1 \rangle \langle m+1 \rangle_2 + \langle m+1 \rangle \langle n+1 \rangle_2)}_{a_1^+ a_2^+ \langle 1 \rangle \langle 1 \rangle} + \sqrt{n+1}\sqrt{m-1} \langle n+1 \rangle \langle m-1 \rangle_2 \\
&\quad + \underbrace{\sqrt{m+1}\sqrt{n-1} \langle m+1 \rangle \langle n-1 \rangle_2}_{a_1^+ a_2^+ \langle 1 \rangle \langle 1 \rangle} + \underbrace{\sqrt{n}\sqrt{m+1} \langle n-1 \rangle \langle m+1 \rangle_2 + \sqrt{m}\sqrt{n+1} \langle m-1 \rangle \langle n+1 \rangle_2}_{a_1^+ a_2^+ \langle 1 \rangle \langle 1 \rangle} \\
&\quad + \sqrt{n}\sqrt{m} (\langle n-1 \rangle \langle m-1 \rangle_2 + \langle m-1 \rangle \langle n-1 \rangle_2)]
\end{aligned}$$

First two & last two terms in $x_1 x_2 |\psi\rangle$ dotted into $\langle \psi |$ will give 0. But the middle 4 terms give possible non zero answers if $n \neq m$ satisfy a relationship:

$$\begin{aligned}
\langle ml_1 \rangle \langle nl_2 \rangle (\langle m+1 \rangle \langle m-1 \rangle_2) &= 1 \text{ if } m=n+1 \Leftrightarrow n=m-1 = 1 \delta_{n,m-1} \\
\langle nl_1 \rangle \langle ml_2 \rangle (\langle m+1 \rangle \langle m-1 \rangle_2) &= 1 \text{ if } m=n-1 \Leftrightarrow n=m+1 = 1 \delta_{n,m+1} \\
\langle ml_1 \rangle \langle nl_2 \rangle (\langle n-1 \rangle \langle m+1 \rangle_2) &= 1 \text{ if } m=n-1 \Leftrightarrow n=m+1 = 1 \delta_{n,m+1} \\
\langle nl_1 \rangle \langle ml_2 \rangle (\langle m-1 \rangle \langle n+1 \rangle_2) &= 1 \text{ if } m=n+1 \Leftrightarrow n=m-1 = 1 \delta_{n,m-1}
\end{aligned}$$

$$\begin{aligned}
\therefore \langle \psi | x_1 x_2 | \psi \rangle &= \frac{\hbar}{4m\omega} [\sqrt{n+1}\sqrt{m} \delta_{n,m-1} + \sqrt{m+1}\sqrt{n} \delta_{n,m+1} + \sqrt{n}\sqrt{m+1} \delta_{n,m+1} + \sqrt{m}\sqrt{n+1} \delta_{n,m-1}] \\
&= \frac{\hbar}{2m\omega} [\sqrt{n+1}\sqrt{m} \delta_{n,m-1} + \sqrt{m+1}\sqrt{n} \delta_{n,m+1}]
\end{aligned}$$

$$\therefore \langle V \rangle^+ = \frac{\hbar^2 A}{m\omega} (n+m+1 - \sqrt{n+1}\sqrt{m} \delta_{n,m-1} - \sqrt{m+1}\sqrt{n} \delta_{n,m+1})$$

(b) Particles have spin $\frac{1}{2}$. We can construct 4 antisymmetric total wavefunctions.

$$\Psi = \begin{cases} \Psi_s(x_1, x_2) \chi_A(s_1, s_2) = \frac{1}{\sqrt{2}} [|n\rangle, |m\rangle_2 + |m\rangle, |n\rangle_2] \frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle] \\ \Psi_A(x_1, x_2) \chi_s(s_1, s_2) = \frac{1}{\sqrt{2}} [|n\rangle, |m\rangle_2 - |m\rangle, |n\rangle_2] \begin{cases} \frac{1}{\sqrt{2}} (|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle) \\ |\downarrow_1\downarrow_2\rangle \end{cases} \end{cases}$$

Notice that the operator is independent of the spin... The expectation value of the symmetric spatial wavefunction is the same as part (a).

Let's concentrate on the antisymmetric spatial wavefn:

$$\begin{aligned} \langle \Psi_A | x_1^2 | \Psi_A \rangle &= \frac{\hbar}{4mw} [\langle n|, \langle m|_2 - \langle m|, \langle n|_2] [n |n\rangle, |m\rangle_2 - m |m\rangle, |n\rangle_2 + (n+1) |n\rangle, |m\rangle_2 - (m+1) |n\rangle, \\ &\quad \uparrow \\ &\quad \text{I have neglected the terms that go to zero.} \end{aligned}$$

$$= \frac{\hbar}{4mw} [n+m+n+1+m+1] = \frac{\hbar}{2mw} [n+m+1] \leftarrow \text{Same as before since minus signs on both the bra \& ket cancel.}$$

$$\text{Similarly } \langle \Psi_A | x_2^2 | \Psi_A \rangle = \frac{\hbar}{2mw} [n+m+1]$$

$$\begin{aligned} \langle \Psi_A | x_1 x_2 | \Psi_A \rangle &= \frac{\hbar}{4mw} [\langle n|, \langle m|_2 - \langle m|, \langle n|_2] (\underbrace{a_1^\dagger a_2^\dagger + a_1^\dagger a_2 + a_1 a_2^\dagger + a_1 a_2}_0) [|n\rangle, |m\rangle_2 - |m\rangle, |n\rangle_2] \\ &= \frac{\hbar}{4mw} [\langle n|, \langle m|_2 - \langle m|, \langle n|_2] [\sqrt{n+1} \sqrt{m} |n+1\rangle, |m-1\rangle_2 - \sqrt{m+1} \sqrt{n} |m+1\rangle, |n-1\rangle_2 + \sqrt{n} \sqrt{m+1} |n-1\rangle, |m+1\rangle_2 \\ &\quad - \sqrt{m} \sqrt{n+1} |m-1\rangle, |n+1\rangle_2] \end{aligned}$$

$$= \frac{\hbar}{4mw} [-\sqrt{n+1} \sqrt{m} \delta_{n,m-1} - \sqrt{m+1} \sqrt{n} \delta_{n,m+1} - \sqrt{n} \sqrt{n+1} \delta_{n,m+1} - \sqrt{m} \sqrt{n+1} \delta_{n,m-1}]$$

$$= -\frac{\hbar}{2mw} [\sqrt{n+1} \sqrt{m} \delta_{n,m-1} + \sqrt{m+1} \sqrt{n} \delta_{n,m+1}]$$

$$\therefore \langle V \rangle = \frac{\hbar \lambda}{mw} [n+m+1 + \sqrt{n+1} \sqrt{m} \delta_{n,m-1} + \sqrt{m+1} \sqrt{n} \delta_{n,m+1}]$$

Notice the change in $\langle V \rangle$ just from symmetrization or antisymmetrization of the spatial wavefunction. This is a demonstration of Hund's 1st rule if energy $\sim f(V)$ where $V = \lambda (x_1 - x_2)^2$.

Problem 4: Although you were asked for the 1st 3 elements, I have gone through the entire Ne configuration.

Z=11: Na: Electron configuration is $(Ne)(3s)$

$$\therefore s = \frac{1}{2}, l = 0, j = \frac{1}{2}$$

$$\therefore ^2S_{\frac{1}{2}} \text{ (No ambiguity so we don't need the Hund Rules).}$$

Z=12, Mg: Elect. config: $(Ne)(3s)^2$

$$\therefore s = 0 \text{ or } 1, 2 \text{ particles w/ } l = 0 \therefore l_{tot} = 0 \\ j = 1 \text{ or } 0.$$

Notice that the $s=1$ state is not allowed. The reason for this is the fact that $l_{tot}=0 \Rightarrow$ the spatial wavefunction is symmetric. The spin part must be antisymmetric (i.e. spin 0 singlet)

$$\therefore ^1S_0$$

Z=13, Al: Elect. config: $(Ne)(3s)^2(3p)$

$$s = \frac{1}{2}, l = 1, j = \frac{3}{2}, \frac{1}{2}$$

Since the $3p$ orbital is less than half filled $J = |L-S| = \frac{1}{2}$.

$$^2P_{1/2}$$

Z=14, Si: Elect. config: $(Ne)(3s)^2(3p)^2$

Again we have two electrons $\therefore s = 0 \text{ or } 1$. Two electrons in $l=1$ gives $l_{tot} = 2, 1, 0$.

1st Hund rule tells us that $s=1$. This is a symmetric wavefunction. We need an antisymmetric spatial wavefunction. Looking at the 1x1 table of Clebsch Gordon Table we see that $l_{tot}=1$ states are antisymmetric.

$$\therefore l_{tot}=1, s=1 \therefore j=2, 1, 0$$

3rd Hund's rule tells us that if the orbital is less than half filled (it is) $J = |L-S| = 0$

$$\therefore ^3P_0$$

Z=15 P: Elect. config: $(Ne) (3s)^2 (3p)^3$

This is the hardest one to figure out. Since we have 3 spin $\frac{1}{2}$ particles.

$s = \frac{3}{2}, \frac{1}{2}$, but 1st Hund's rule tells us that s must be $\frac{3}{2}$.

What are the possibilities for l_{tot} ? Since we have 3 $l=1$ particles, so l_{tot} could be 3, 2, 1, 0. We need to use the 2nd Hund's rule to figure out which one is the lowest energy. Recall that the 2nd Hund's rule tells you that the highest l state that satisfies the Pauli Exclusion principle is the lowest energy.

$s = \frac{3}{2}$ is a symmetric spin state (all spins pointing up) \therefore the spatial wave function must be antisymmetric.

When we combine two particles w/ $l=1$, we get $l=2, l=1, l=0$

- We choose the $l=1$ for the 2 particles & combine this with the 3rd $l=1$ particle.

$l=1$ 2 particles: $|11\rangle = \frac{1}{\sqrt{2}} [|11\rangle |10\rangle - |10\rangle |11\rangle]$
 $|10\rangle = \frac{1}{\sqrt{2}} [|1+1\rangle |1-1\rangle - |1-1\rangle |1+\rangle]$
 $|1-1\rangle = \frac{1}{\sqrt{2}} [|10\rangle |1-1\rangle - |1-1\rangle |10\rangle]$

} using CG Table

Now using CG Table combine these with $l=1$ state & look at the symmetries.
 $|12,2\rangle = |11\rangle |11\rangle + \frac{1}{\sqrt{2}} [|11\rangle |10\rangle - |10\rangle |11\rangle] |11\rangle = \frac{1}{\sqrt{2}} [|11\rangle |10\rangle |11\rangle - |10\rangle |11\rangle |11\rangle]$ ← This state
 ↑
 2 particle is neither symmetric nor antisymmetric under particle exchange.

$$|12,1\rangle = \frac{1}{\sqrt{2}} [|11\rangle |10\rangle + |10\rangle |11\rangle] = \frac{1}{\sqrt{2}} [\frac{1}{\sqrt{2}} (|11\rangle |10\rangle - |10\rangle |11\rangle) |10\rangle + \frac{1}{\sqrt{2}} (|11\rangle |1-1\rangle - |1-1\rangle |11\rangle) |11\rangle]$$
 $= \frac{1}{2} [|11\rangle |10\rangle |10\rangle - |10\rangle |11\rangle |10\rangle + |11\rangle |1-1\rangle |11\rangle - |1-1\rangle |11\rangle |11\rangle]$ ← also has no symmetry.

You can see that none of the $l_{\text{tot}}=2$ states are going to have symmetry.

Let's try $l_{\text{tot}}=1$ states.

$$|11\rangle = \frac{1}{\sqrt{2}} [|11\rangle |10\rangle - |10\rangle |11\rangle] = \frac{1}{\sqrt{2}} [\frac{1}{\sqrt{2}} (|11\rangle |10\rangle - |10\rangle |11\rangle) |10\rangle - \frac{1}{\sqrt{2}} (|11\rangle |1-1\rangle - |1-1\rangle |11\rangle) |11\rangle]$$
 $= \frac{1}{2} [|11\rangle |10\rangle |10\rangle - |10\rangle |11\rangle |10\rangle - |11\rangle |1-1\rangle |11\rangle + |1-1\rangle |11\rangle |11\rangle]$ ← again, no symmetry under particle 2 & 3 exchange.

so spin 1 states have no symmetry either.

Let's try spin 0:

$$\begin{aligned} |100\rangle &= \frac{1}{\sqrt{3}} [|111\rangle|11-1\rangle - |110\rangle|100\rangle + |1-1\rangle|111\rangle] \\ &= \frac{1}{\sqrt{3}} \left[\frac{1}{\sqrt{2}} (|111\rangle|110\rangle - |110\rangle|111\rangle) |11-1\rangle - \frac{1}{\sqrt{2}} (|111\rangle|11-1\rangle - |11-1\rangle|111\rangle) |100\rangle + \frac{1}{\sqrt{2}} (|110\rangle|11-1\rangle - |11-1\rangle|110\rangle) |111\rangle \right] \\ &= \frac{1}{\sqrt{6}} \left[|11\rangle|110\rangle|11-1\rangle - |110\rangle|111\rangle|11-1\rangle - |111\rangle|11-1\rangle|10\rangle - |11-1\rangle|111\rangle|110\rangle + |110\rangle|11-1\rangle|111\rangle - |11-1\rangle|110\rangle|111\rangle \right] \end{aligned}$$

If you stare at this for a while you see that $|100\rangle$ state is antisymmetric when...

∴ Only allowed l_{tot} state is 0.

So $s = \frac{3}{2}$, $l = 0$, & by the $3^{\pm 2}$ rule $j = |l-s| = \frac{3}{2}$ since orbital is no more than half filled.

∴ ${}^4S_{3/2}$

Z=16 S: Elect. config (Ne) $(3s)^2 (3p)^4$

Here is the trick for orbitals more than half full: You can think of them as closed shells with holes in them. For example, for $Z=16$, we have $(3p)^4$. You could look at this as: 4 electrons in the p orbital or a p orbital with 2 holes ("anti-electrons"). As a result, $(3p)^4$ is rather similar to $(3p)^2$ $s=0$ or 1 but 1st Hund's rule tells us that it must be 1 ← symmetric.

∴ The total l state must be symmetric. We have 2 holes with $l=1$ ∴ the possible values for $l_{tot} = 2, 1, 0$. $l_{tot}=2$ and 0 are symmetric and $l_{tot}=1$ is antisymmetric as we have seen. ∴ $l_{tot}=1$ ← antisymmetric to obey Pauli Exclusion principle.

How about J ? Since the orbital is more than half filled, we want $J=l+s=2$.

∴ 3P_2

Z=17 Cl: Elec Config: $(Ne)(3s)^2(3p)^5$ ← can think of it as 1 hole.

$$\therefore s = 1/2 \quad l = 1 \quad j = l+s = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\therefore {}^2P_{3/2}$$

Z=18 Ar: Elec Config: $(Ne)(3s)^2(3p)^6$ ← the shell is closed $\therefore s = 0$

$$\therefore {}^1S_0$$

Problem 5

(a) Boron ($Z=5$) : $(1s)^2(2s)^2(3p)^1$

Carbon ($Z=6$) = $(1s)^2(2s)^2(3p)^2$

Nitrogen ($Z=7$) = $(1s)^2(2s)^2(3p)^3$

(b) For Boron, one electron in p state $\therefore l_{tot}=1$

For Carbon, we have two electrons in p state. $\therefore l_{tot}=2, 1, 0$

For Nitrogen, " " three " " " $\therefore l_{tot}=3, 2, 1, 0$.

(c) No ambiguity for Boron.

. Carbon: $s=1$ or 0 . 1st Hund's rule tells us that $s=1$ ← symmetric

∴ we need antisymmetric spatial wavefunction. As we have seen in Problem 4 $l_{tot}=1$ is antisymmetric

$$\therefore \boxed{l_{tot}=1}$$

. Nitrogen: $s=3/2$ by 1st Hund's Rule. As we have seen in Problem 4 when we combine 3 $l=1$ particles, the only antisymmetrized state is $|000\rangle$ (there are no symmetric states either).

$$\therefore \boxed{l_{tot}=0}$$